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DETERMINISM AND PREDICTABILITY

ABSTRACT. Theoretical determinism, as it is usually ascribed to Laplace, is neither verifiable nor falsifiable and has therefore no real content. It is not the same as predictability of actually observable phenomena. On the other hand, predictability is not an abstract principle; rather it is true to a certain degree, depending on the phenomena considered. It can be discussed only by examining the scientific state of affairs. This is done in some detail for classical statistical mechanics. Much of a recently published debate on determinism (Amsterdamski et al. 1990) is thereby obviated.

1. INTRODUCTION

Recently a debate on determinism has been published consisting of a number of contributions of various authors.[1] The opening gambit is an article by R. Thom, in which he asserts that determinism is a prerequisite of the very existence of science and, therefore, the ideal to which scientists should strive. Anyone who introduces chance is a betrayer of this ideal and a deserter. One particular target of Thom is J. Monod, who considered chance as an essential element of biological evolution (Monod 1970). The other target is Prigogine and his school (Prigogine and Stengers 1980), who consider random fluctuations as the source from which ordered structures arise.

The present article is an attempt to clarify a number of physical *facts* which in the heat of the debate did not receive the prominence that facts deserve.[2] Actually I have the impression that an explicit formulation of these facts renders much of the debate futile. Quotations from ancient philosophers cannot replace facts.

In the published debate quantum mechanics is rarely mentioned. Accordingly I shall also ignore it until the very end. Of course, that makes the discussion rather academic, but it will appear at the end that this shortcoming is less serious than might be expected.

2. LAPLACIAN DETERMINISM

For a meaningful debate it is necessary to define the terms and then stick to the definitions. Almost all of the contributors to the present

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debate agree with the description of determinism given by Laplace (Laplace 1814). He imagined an 'intelligence' who at a given instant in time knows the state of all elements of which the universe is composed and, moreover, has an unlimited capacity for computing. Such a demon[3] would have the entire future and the entire past before his eyes. – It is clear that Laplace had the motion of the planets before his eyes, and extrapolated to a universe composed of interacting small bodies. As a description of the basic property of differential equations his statement is still perfectly correct.[4]

As a definition of physical determinism, however, it cannot serve, because there is no way of testing whether or not the real world is in agreement with it. The postulated demon cannot communicate with us, nor make his existence felt in any way, since that would mean that he influences our observed world and thereby violates the supposed determinism. He cannot act in any direct or indirect way on our experience. That is tantamount to saying that he does not exist.

Some authors politely call this idea of Laplace 'theoretical' or 'ontological' determinism, but it does not seem to be unfair to regard this definition as having no content that can be related to the observed world.

THEOREM: The ontological determinism à la Laplace cannot be proved or disproved on the basis of observations.

To show that it cannot be disproved, suppose I live in a free world A, in which I am able to deduce from my observations that the events in A are not determined. It may be that they are subject to occasional random jumps or that I am able to prove that I have a free will; at any rate no unique connection linking successive states of A can be found. Now imagine a second world B, which is fully determined, namely by the following rule: everything in B is an exact copy of how A looked one million years earlier. Everyone in B has exactly the same experience as his prototype in A. He has therefore the same right as his prototype to deduce that his world is not deterministic. But he would be wrong. It follows that the way in which the prototype in A proved the absence of determinism was also wrong. After all, for all he knows, A might be the replica of a still earlier world.

On the other hand, ontological determinism cannot be proved because we live in a single universe having a single sequence of successive states. It is therefore not possible for us to say that one state determines another, no more than, for instance, in the sequence of prime numbers one prime determines the next one. The counterargument that maybe the demon can see the entire future is irrelevant as we cannot communicate with that demon. This proves the theorem.

3. PREDICTABILITY

Consequently, if the word 'determinism' is to serve a useful purpose, a more concrete definition is required. In fact, most of the contributors to the debate, having paid lip service to Laplace, almost unnoticeably substitute for his demon a human observer. They thereby reduce determinism to predictability, i.e., the question whether an actual observer, a biologist or a physicist, is able to predict future events. This reduction of Laplacian determinism to actual predictability is a drastic step. On the one hand, it brings the question from the philosophical clouds down to earth, where one may hope to find an answer. On the other, it is reduced to a technical question about the the state of affairs in the relevant science.[5]

Science, unlike mathematics, is forced to deal with incomplete and imprecise data.[6] Laplace's demon sees *all* chromosomes, and is able to compute the motion of *all* molecules in the biological cell and thereby predict the outcome of every single meiosis, but he cannot tell it to us.[7] The biologist, however, is lucky to have Mendel's laws, even though they deal only with statistical averages. Is he therefore a betrayer of the scientific ideal? Should he be called a deserter because he has given up hope of ever being able to compute the outcome of the molecular motion, which he cannot observe anyway? On the contrary, he is perfectly justified scientifically to appeal to probability theory as a means to extract useful information from incomplete data.[8]

4. PREDICTABILITY IN PHYSICS

In physics the state of affairs is the same, but it can be formulated in more detail. Ever since Boltzmann, a gas is considered as a collection of molecules, which move according to deterministic equations of motion. A human observer, however, cannot see the individual molecules and cannot hope to solve their equations of motion. His observations are on a much coarser level, where many molecules are lumped together into a fluid element; the element is then described by only a few variables such as mass, average velocity, and total energy. Experience tells us that, amazingly, this restricted set of variables is again governed by certain differential equations, such as the hydrodynamic equations.

Statistical mechanics therefore starts from an underlying microscopic world governed by deterministic differential equations, like the world of Laplace. On a coarse-grained macroscopic level the same system can again be described by deterministic differential equations. These are the equations that Thom has in mind. However, they are only approximate: the actual values of the macroscopic variables deviate by small, irregularly varying amounts, called fluctuations. To find the precise values of these deviations requires the impossible task of solving the microscopic equations exactly. The physicist is lucky that in the absence of an exact description it is possible to say something about the averaged fluctuations and their statistics.

To summarise, although mathematically the macroscopic equations describe a deterministic motion, the physical system they purport to describe deviates from this motion by random fluctuations and is therefore not deterministic. Laplacian determinism belongs to the microscopic level alone.[9]

These are the basic facts of statistical mechanics. It is hard to believe that Thom is aware of them, considering that he quotes as a prime example of Laplacian determinism the equation $pV = RT$; although it is manifestly macroscopic, approximate, and involves the pressure and the temperature, which exist only as statistical averages.

5. INSTABILITIES

In most cases the fluctuations are so small that in practice they can be ignored and the macroscopic equations predict the behaviour with great accuracy. Technology is based on this approximation. In some cases, however, the macroscopic equations do not themselves uniquely determine the successive states of the system. This happens when, on varying an external parameter such as the temperature or the temperature gradient of the surroundings, one arrives at a bifurcation point of the macroscopic equations. For systems in equilibrium these are the phase transition points. For systems whose environment keeps them forcibly in a stationary non-equilibrium state (the trade name is 'open systems'), this bifurcation underlies the Bénard cells and similar phenomena.

Such a bifurcation in the macroscopic equations means that the macroscopic approximation is able only to tell that, while the parameter varies, the system will follow either one or the other branch of the fork. The decision about which one will actually be followed then resides in the next approximation, i.e., the fluctuations. The reason why they are able to influence the macroscopic behaviour is that the macroscopic equations themselves are unstable at the point of bifurcation.

However, the precise deviations are not known to us but merely their statistics. Hence, we are unable to predict which branch will be taken; only a probability can be computed for each branch. In this situation therefore even the approximate predictability on the macroscopic level breaks down. A pencil standing on its tip will fall down according to the laws of classical mechanics, but the direction in which it falls is determined by the random fluctuations, which are needed to break the symmetry.

In fluids and other systems whose macroscopic equations are homogeneous in space, it may happen that one or more branches correspond to a macroscopic state that is not homogeneous and therefore breaks the symmetry. To clothe this in the slogan 'Order out of Chaos' used by Prigogine and Stengers (Prigogine and Stengers 1980) is inappropriate for the following reasons. The word 'Chaos' refers to the fluctuations, but they are negligible in the inhomogeneous state; they are merely instrumental at the bifurcation point in choosing which particular branch is going to be followed. The word 'Order', which is here applied to a spatial structure such as the Bénard cells, is purely intuitive without well-defined meaning.

6. SUMMARY OF STATISTICAL MECHANICS[10]

First, the starting point is the microscopic world of molecules governed by deterministic differential equations of motion. These equations are taken to be exact. They are invariant for time reversal.

Second, the following procedure, not fully understood, gives a 'mesoscopic' description. One selects a small number of coarse-grained variables and eliminates all others by means of an appropriate randomness assumption (called '*Stosszahlansatz*', 'molecular chaos', or 'random phase'). The result is that the evolution of the coarse-grained or macroscopic variables is described as a stochastic process. The time-dependent probability is governed by the 'master equation'. It is irreversible; the time symmetry has been broken by the randomness assumption.

Third, from the mesoscopic level one arrives at the macroscopic description in the limit of large system size. When the size tends to infinity the fluctuations become relatively small, and it is therefore possible to extract from the master equation a set of differential equations which provide a deterministic description of the motion of the macroscopic variables. The next term in the expansion gives the fluctuations, which turn out to be Gaussian. The higher terms represent corrections to the Gaussian character (van Kampen 1981). It is incorrect to regard the macroscopic equations as the basic description of the system, with fluctuations tagged on as perturbations, with the ad hoc assumption that they are Gaussian.[11]

Fourth, concepts like instability and bifurcation belong to the macroscopic limit – just as in equilibrium statistical mechanics one needs the thermodynamic limit for defining phase transitions. The fluctuations that occur in finite systems have the effect of *blurring* the transitions and bifurcations. To say that they *shift* the transition points, or even create new transitions, makes no sense unless one redefines this term in an artificial way.[12]

Fifth, the crucial problem of statistical mechanics is the justification of the randomness assumption that is needed for going from the reversible deterministic microscopic equations to the irreversible stochastic mesoscopic description. This problem has not been solved, but it is clear that a necessary ingredient is that the microscopic equations have solutions that depend very sensitively on the initial data. This sensitivity, together with the coarseness of observations, is the way in which deterministic equations can give rise to a probabilistic appearance, as was known to Gibbs (Gibbs 1902), Poincaré (Poincaré 1908, chap. 4) and others.

Sixth, this sensitive dependence on the initial data has been popularised through the study of chaos (used here in the precise mathematical sense). It is not correct to say that it is at variance with Laplacian determinism, because Laplace's demon was able to observe and compute with infinite precision. But it is true that it affects the predictability of systems governed by differential equations, as, for instance, in meteorology. How important this effect is depends on the system and on the technical possibilities of computers. It is hard to see how this can have any bearing on determinism as a philosophical principle.

Seventh, as argued above, randomness in physical systems is caused by the fact that the elimination of the many microscopic variables is not exact but leaves the fluctuations as a residue. It has nothing to do with ignorance of the precise initial state[13] but only with my inability to follow the microscopic motion.[14] Obviously, the random motion of the Brownian particle under my microscope is *not* a consequence of my ignorance of initial data at some initial time, but of my inability to predict the behaviour of the surrounding molecules.

Eighth, an appeal to the initial state would not even be enough. The master equation describes a Markov process, which means that at each instant the future probability distribution is unaffected by the past. To achieve this semigroup character of the mesoscopic description it is necessary that the randomness assumption in point two be made at a sequence of successive instants. This *repeated randomness assumption* is very drastic and its justification is by no means clear, but at any rate it cannot be explained by ignorance of the initial data.

Ninth, *quantum mechanics* radically alters the microscopic foundation. Roughly speaking, one may say that it causes the phase space to break up in grains of the order of Planck's constant h (per degree of freedom). Yet this has little effect on the mesoscopic level because these fine grains are much smaller than the coarse grains of the observer. Also, the randomness due to the elimination of microscopic variables overshadows the lack of determinism inherent in the quantum mechanical foundation (van Kampen 1954). Consequently, the mesoscopic and the macroscopic pictures remain virtually the same; only the actual values of the coefficients are affected by the quantum mechanics.

Last, it has been claimed that quantum mechanics is deterministic after all since the Schrödinger equation uniquely determines the future wave function from its present form.[15] But the wave function is merely a mathematical tool which serves to compute the probability of observed events.[16] One may compute the wave function of a radioactive nucleus, but one cannot predict when it will decay. The determinism of the wave function is as elusive as that seen by Laplace's demon.

NOTES

[1] Amsterdamski et al. 1990. Actually most of the contributions have appeared before, some almost ten years ago. A reputable publisher ought to have made this clear rather than leave it to the buyer to glean this information from an occasional indiscreet footnote.

[2] H. Atlan, (in Amsterdamski et al. 1990, p. 113) feels that the cultural ambiance along the Seine is responsible.
[3] The term 'demon' in analogy with Maxwell's demon is not used by Laplace, but is more convenient than his 'intelligence'.
[4] Laplace was thinking of second-order differential equations and accordingly, when he talks about a 'state', it consists of the coordinates *and* velocities of all bodies. His assertion would be untrue if the word 'state' denoted the coordinates alone. On the other hand, if the differential equations were of third order, the assertion would still be true provided one includes the instantaneous values of the accelerations as belonging to the 'state'. It would still be true for bodies with electromagnetic interactions provided one includes the electromagnetic field or, alternatively, the entire past history of the bodies. Yet the assertion is not a tautology if one interprets it as meaning that the future is uniquely determined by the past.
[5] Amsterdamski (in Amsterdamski et al. 1990, p. 231) does make the distinction between ontological determinism and predictability, but he does not reject the former as meaningless.
[6] This was forcefully argued by Duhem (Duhem 1906) but does not appear to satisfy some modern luminaries (Bell 1990).
[7] "Every sexual act is a lottery", says E. Morin (in Amsterdamski et al. 1990, p. 97); this is true for the human observer, but not in the eyes of the demon.
[8] One may again quote Laplace, who called probability theory 'common sense reduced to computation' and "le supplément le plus heureux, à l'ignorance et à la faiblesse de l'esprit humain".
[9] The determinism of the molecular motion is similar to, but not identical with, that of Laplace. The motion is inevitably perturbed by the outside world, while Laplace's demon encompasses the entire universe. Amsterdamski distinguishes local and global determinism (in Amsterdamski et al. 1990, p. 231). The perturbations affect the microscopic motion but not the macroscopic behaviour, as pointed out by D. Ruelle (in Amsterdamski et al. 1990). In quantum mechanics this fact is important for the theory of measurement (van Kampen 1988, p. 97). It is also the reason why it is permissible to represent in statistical mechanics a system as if it were fully isolated.
[10] This section is more technical and also contains some ideas that are not yet universally accepted.
[11] This view is so widespread among physicists that one can hardly blame Thom for not knowing that fluctuations are a consequence of the underlying microscopic equations (see Amsterdamski et al. 1990, p. 144). We are talking about *internal* fluctuations, not those that are impressed on the system by some outside agency, such as a noise generator in an electric circuit.
[12] Prigogine in Amsterdamski et al. 1990, p. 104; also Horsthemke and Lefever, 1985; criticised by Thom in Amsterdamski et al. 1990, p. 144.
[13] This is often taken for granted. For instance, in derivations of the master equation (Van Hove 1955; and 1957) and of the Langevin equation (Zwanzig 1960), and in the linear response theory (Kubo 1957).
[14] The word 'inability' is too strong. Even when I am able to follow the individual particles, such as asteroids, I may still prefer an average description, because it is simpler and suffices for many purposes.
[15] I. Ekeland in Amsterdamski et al. 1990, p. 165.
[16] By the same token one might claim that Brownian motion is deterministic since it may be described by the Fokker-Planck equation.

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