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A New Indirect Method of Determining Density of Cometary Nuclei

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ABSTRACT

A new indirect method for determining the bulk density of cometary nuclei is presented. This method has been applied to 136 comets taken from the literature. The method uses the calibration function in the form $a_1 = 10.88526 + 3.39275 \times a_2$ and the equation $\log D_{\rm N} = a_2 - 0.13H$, where $D_{\rm N}$ is the nucleus' diameter [km], and H is the comets' absolute total magnitude (coma + nucleus). To obtain the nuclear bulk density [kg/m^3] the formula $d = \frac{6}{\pi} \times 10^a$ where $a = a_1 - 3a_2 - 9$ is used. The mean nuclear bulk density of the 136 comets is 520 ± 10 kg/m^3. The relationship between the nuclear bulk density and the origin of the comets is found. For the 40 long-period comets the mean density is 470 ± 10 kg/m^3, whereas for the 94 short-period comets from Jupiter family it is 540 ± 10 kg/m^3. The accuracy of the calibration function is 1.5%.

Key words: *Methods: data analysis – Comets: general*

1. Introduction

Determining the density of a comet's nucleus is a much more difficult task than determining its brightness and size. For several decades, attempts have been made to determine the density of comet nuclei. For this purpose, various methods and techniques were used. The first attempt to estimate the density of cometary nuclei was made by Donn (1963). In this method realistically packed snow from Alpine glaciers was used. He obtained a density of 0.48 ± 0.15 g/cm^3. Delsemme and Wenger(1970), using in their technique laboratory snow cluster formation, estimated the density in the range of 0.33–0.54 g/cm^3. Sagdeev *et al.* (1988), using the method of H_2O out-gassing for comet 1P/Halley, obtained its nuclear density 0.28–0.65 g/cm^3. However, Rickman (1989) obtained a density of 0.6 $(+0.9, -0.4)$ g/cm^3 for the comet's nucleus using the same method. Sekanina (1991), using a Jacobi ellipsoid of comet 10P/Tempel 2, obtained a nuclear density of 0.54 g/cm^3. Luu and Jewitt (1992), using the centrifugal rotation of comet

31P/Schwassmann-Wachmann 2, estimated a density of its nucleus higher than 0.46 g/cm^3. Prialnik and Bar-Nun (1992), based on the outburst of comet 1P/Halley at a distance of 14.3 au, obtained a density of its nucleus equal to 0.5 g/cm^3. Boss (1994), using the model of the disruption of comet Shoemaker-Levy 9, estimated its nucleus density as lower than 0.7–1.5 g/cm^3. However, Asphaug and Benz (1994), based on the tidal breakup of a non-rotating nucleus, obtained a nuclear density of 0.5 g/cm^3. Solem (1994, 1995), and Asphaug and Benz (1996), based on the tidal disruption of comet Shoemaker-Levy 9, obtained the nuclear densities of 0.5, 0.55, and 0.6 g/cm^3, respectively. Samarasinha and Belton (1995), using the numerical simulation of the rotational state of the comet nuclei, estimated a nuclear density lower than 0.4 g/cm^3. Meech (1996)[1] and Meech *et al.* (1997), based on the Hubble Space Telescope observations of 95P/Chiron's atmosphere, estimated the density of the nucleus to be lower than 1.0 g/cm^3. Harmon *et al.* (1999), based on the radar observations of comet C/1983 H1 (IRAS-Araki-Alcock), obtained a nuclear density of 0.45 g/cm^3. Farnham and Cochran (2002), from the non-gravitational acceleration of comet 19P/Borrelly, estimated a nuclear density of 0.49 $(+0.34, -0.20)$ g/cm^3. However, Davidsson and Gutiérrez (2004), using the method of H_2O out-gassing for the comet, obtained its nuclear density in the range of 0.18–0.30 g/cm^3. Hsieh *et al.* (2004), using the centrifugal rotation of comet 133P/Elst-Pizarro, estimated the density of its nucleus to be higher than 1.3 g/cm^3. Snodgrass *et al.* (2005), using the above method, obtained a nuclear density of comet 92P/Sanguin higher than 0.49 g/cm^3. A'Hearn *et al.* (2005), based on the expansion of the base of conical ejecta from the impactor of comet 9P/Tempel 1, estimated a nuclear density of 0.62 $(+0.47, -0.33)$ g/cm^3. Harmon and Nolan (2005), based on radar observations of comet 2P/Encke, obtained a nuclear density in the range of 0.5–1.0 g/cm^3. Davidsson and Gutiérrez (2006), based on the water production and non-gravitational forces of comet 81P/Wild 2, estimated its nuclear density to be lower than 0.58 g/cm^3. Rosetta's mission to comet 67P/Churyumov-Gerasimenko revealed the exact density of its nucleus to be 532 ± 7 kg/m^3 (Jorda *et al.* 2016). The mean nuclear density of 31 comets, equal to 453 ± 29 kg/m^3, has previously been determined by a new method, and additionally a nuclear bulk density of 556 ± 45 kg/m^3 was determined using the same method for 29P/Schwassmann-Wachmann 1 (Paradowski 2020). This method cannot determine a bulk density for each nucleus separately, with the exception of 29P/Schwassmann-Wachmann 1. The above methods for determining the density of the nuclei are limited to a few comets.

In order to determine the bulk density for more cometary nuclei, we have developed a new indirect method using known nuclei sizes and the absolute total magnitudes (coma plus nucleus) of comets. We applied this method to 136 comets taken from the literature.

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[1] *http://www.ifa.hawaii.edu/ meech/papers/acm96.pdf*

2. The Method

The indirect method uses a calibration function prepared from the known bulk densities of some comet nuclei, and the derived formula for nuclear bulk density [kg/m^3] has the form given in Paradowski (2020) as

$$d = \frac{6}{\pi} \times 10^a, \tag{1}$$

where $a = a_1 - 3a_2 - 9$.

Parameters a_1 and a_2 occur in Eqs.(2) and (3) given by Paradowski (2020):

$$\log M = a_1 + b_1 H, \tag{2}$$

and

$$\log D_\mathrm{N} = a_2 + b_2 H, \tag{3}$$

where M is the mass of the comet [kg], D_N is the effective diameter of its nucleus [km], b_1 and b_2 are the slope coefficients [mag^{-1}], and H is the absolute total magnitude of the comet (coma plus nucleus). The absolute total magnitude H has the form:

$$H = m_1 - 5\log\Delta - 2.5n\log r, \tag{4}$$

where m_1 is the apparent visual magnitude of the comet, r and Δ are the heliocentric and geocentric distances [au], respectively, and n is the photometric index. The slope coefficients b_1 and b_2 do not change with the nuclear bulk density, as we have shown with the example of 20 comets taken from the literature (see Figs. 1 and 2). The graphs of $\log D_\mathrm{N}$ to H (Fig. 1) and $\log M$ to H (Fig. 2) show that the nuclei of the same bulk density lie on straight lines fitted with Eqs.(3) and (2), respectively. The two parallel lines of fit correspond to nuclear bulk densities of 500 kg/m^3 and 620 kg/m^3, *i.e.,* nucleus densities 81P/Wild 2 (Davidsson and Gutiérrez 2006) and 9P/Tempel 1 (A'Hearn *et al.* 2005), respectively. The open circles concern comets 67P/Churyumov-Gerasimenko and 2I/Borisov with nuclear densities of 532 kg/m^3 (Jorda *et al.* 2016) and 500 kg/m^3 (Jewitt *et al.* 2020), respectively, and are not included in the fit. We can see that 2I/Borisov lies on the fitting line, corresponding to a nuclear density of 500 kg/m^3. Thus, we assume that $b_1 = -0.39$ and $b_2 = -0.13$ for all studied comets. The same values of b_1 and b_2 were obtained by Sosa and Fernández (2011) for 9 LP comets, they assumed a nuclear bulk density of 400 kg/m^3.

In a previous paper (Paradowski 2020), we noticed that the values of the parameters a_1 and a_2 increase with the bulk density of comet nuclei, while the slope coefficients b_1 and b_2 do not depend on this density. In order to find a linear regression of the a_1 and a_2 parameters, we selected three comets with exactly known nuclear densities. These included comet 67P/Churyumov-Gerasimenko, which has the most accurately measured nucleus density from the Rosetta mission of 532 ± 7 kg/m^3 (Jorda *et al.* 2016), 9P/Tempel 1 with a fairly well measured

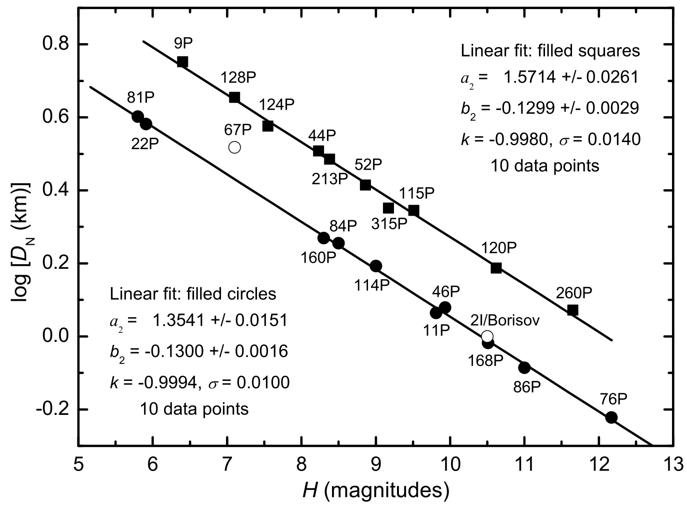

Fig. 1. Logarithm of the effective nuclear diameter D_N [km] as a function of the absolute total magnitude (coma + nucleus) H for 20 selected comets from the literature (see Table 1). The figure shows linear regressions with correlation coefficients k and standard deviations σ, and the determined parameters a_2 and b_2 from the least-squares fit (Eq. 3).

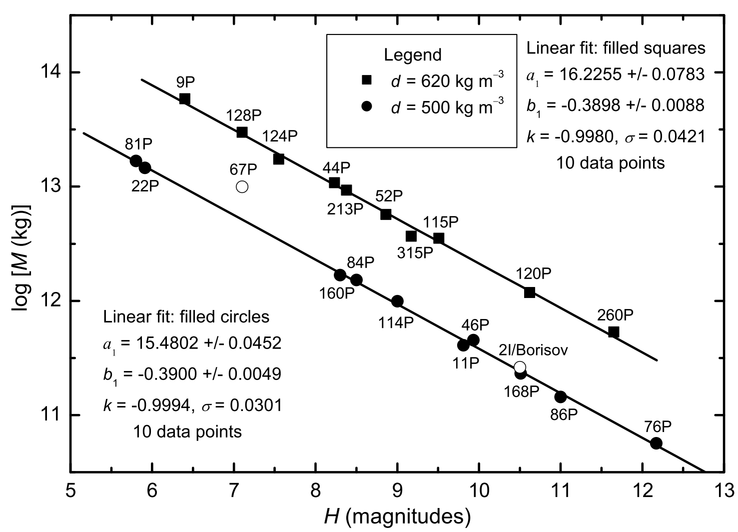

Fig. 2. Logarithm of the mass M [kg] as a function of the absolute total magnitude (coma + nucleus) H for the same 20 comets as in Fig. 1. The figure shows linear regressions with correlation coefficients k and standard deviations σ, and the determined parameters a_1 and b_1 from the least-squares fit (Eq. 2).

density of its nucleus from the Deep Impact mission of 620 kg/m^3 (A'Hearn *et al.* 2005), and 81P/Wild 2 with an assumed density of its nucleus observed by the Stardust mission equal to 500 kg/m^3 (Davidsson and Gutiérrez 2006). In addition, we add 31 comets for which the mean bulk density was 453 ± 29 kg/m^3 (Parad-

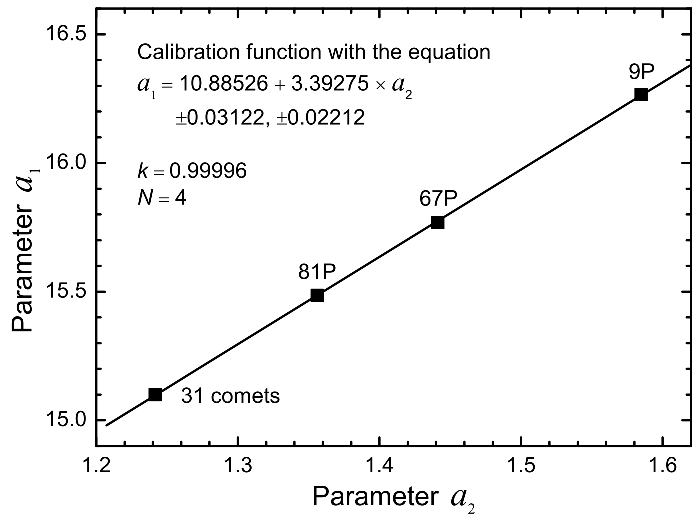

Fig. 3. Linear correlation between the a_1 and a_2 parameters for the four different bulk densities of the comet nuclei. The densities are 453 kg/m^3 as the mean of 31 comets given in Paradowski (2020), 500 kg/m^3 for comet 81P/Wild 2 (Davidsson and Gutiérrez 2006), 532 kg/m^3 for 67P/Churyumov-Gerasimenko (Jorda *et al.* 2016), and 620 kg/m^3 for 9P/Tempel 1 (A'Hearn *et al.* 2005).

owski 2020). Therefore, we have four data points whose coordinates a_1 and a_2 may be computed from Eqs.(2) and (3) after substituting the nuclear bulk density converted to mass M [kg], effective nuclear diameter $D_{\rm N}$ [km], $b_1 = -0.39$, $b_2 = -0.13$, and absolute magnitude (coma + nucleus) H for these three comets. We obtain $a_1 = 15.76866$ and $a_2 = 1.44125$ when $D_{\rm N} = 3.298$ km (Jorda *et al.* 2016) and $H = 7.1$ mag (COBS[2]) for comet 67P/Churyumov-Gerasimenko, $a_1 = 16.26584$ and $a_2 = 1.58482$ when $D_{\rm N} = 5.66$ km (Thomas *et al.* 2013a) and $H = 6.4$ mag (Ferrín 2009)[3] for 9P/Tempel 1, and $a_1 = 15.48615$ and $a_2 = 1.35606$ when $D_{\rm N} = 4.0$ km (Meech *et al.* 2004) and $H = 5.8$ mag (Ferrín 2009) for 81P/Wild 2. Then $a_1 = 15.1$ and $a_2 = 1.24151$ for the 31 comets we take from the previous paper (Paradowski 2020). The four data points and the linear regression for these points with a correlation coefficient $k = 0.99996$ are shown in Fig. 3. The solid straight line that fits the four data points with the equation

$$a_1 = 10.88526 + 3.39275 \times a_2 \tag{5}$$

is called the calibration function. From the least-squares fitting, we obtain two parameters with their errors of 10.88526 ± 0.03122 and 3.39275 ± 0.02212, and a standard deviation $\sigma = 0.00554$. The above calibration function is the basis for determining the bulk density of cometary nuclei. Therefore, if the value of parameter a_2 is known, then the value of parameter a_1 can be calculated from the calibration function given in Eq.(5). The value of parameter a_2 can easily be

[2]Observations from the COBS International Database *https://www.cobs.si*
[3]Atlas of Secular Light Curves of Comets. *https://arxiv.org/ftp/arxiv/papers/0909/0909.3498.pdf*

determined from Eq.(3), substituting the nucleus diameter value [km] $D_{\rm N}$, $b_2 = -0.13$, and the absolute total magnitude of the comet (coma + nucleus) H. Thus, knowing the values of parameters a_1 and a_2 we can easily determine the bulk density of the comet nucleus d from Eq.(1).

The accuracy of our indirect method for determining the bulk density of cometary nuclei can be estimated from the calibration function given in Eq.(5). For this purpose, we use the known nuclear bulk density of comet 67P/Churyumov-Gerasimenko, equal to 532 ± 7 kg/m^3 (Jorda *et al.* 2016), and calculate from Eqs.(1), (3) and (5) the bulk density for the comet nucleus. Substituting the absolute total magnitude of the comet (coma + nucleus) $H = 7.1$ (COBS) and the effective diameter of its nucleus $D_{\rm N} = 3.298 \pm 0.014$ km (Jorda *et al.* 2016) into Eq.(3) we obtain the value of the parameter a_2 equal to 1.44125, and then from Eq.(5) we determine the parameter a_1 equal to 15.77506. Finally, from Eq.(1) we obtain a bulk density of 540 kg/m^3. The density difference of 8 kg/m^3 gives the estimate of the accuracy of 1.5%.

3. Results

Once the values of the effective nuclear radius $r_{\rm N}$ and absolute total magnitude (coma + nucleus) H of the comet are known from the literature, then we can determine the nuclear bulk density d using Eq.(3), and the calibration function with Eq.(5), and Eq.(1). The determined nuclear bulk density d of 96 short-period (SP) comets and 40 LP comets is presented in Tables 1 and 2. In the text below, we compare the bulk density of cometary nuclei determined using the new indirect method with some determinations given in the literature.

1. 1P/Halley: The determined nuclear bulk density of 540 ± 70 kg/m^3 is consistent with 556 ± 45 kg/m^3 (Paradowski 2020). However, Prialnik and Bar-Nun (1992) obtained a slightly lower density of 500 kg/m^3, and Rickman (1989) estimated a density of 600 $(+900, -400)$ kg/m^3.

2. 2P/Encke: The determined nuclear bulk density of 660 ± 110 kg/m^3 is lower than the 800 kg/m^3 estimated by Sosa and Fernández (2009). However, Harmon and Nolan (2005) obtained a density in the range of 500–1000 kg/m^3, which includes our result.

3. 10P/Tempel 2: The determined nuclear bulk density of 550 ± 80 kg/m^3 is consistent with 540 kg/m^3 obtained by Sekanina (1991).

4. 19P/Borrelly: The determined nuclear bulk density of 440 ± 60 kg/m^3 is close to 490 $(+340, -200)$ kg/m^3 estimated by Farnham and Cochran (2002).

5. 29P/Schwassmann-Wachmann 1: The determined nuclear bulk density of 540 ± 100 kg/m^3 is consistent with 556 ± 45 kg/m^3 (Paradowski 2020).

However, Kossacki and Szutowicz (2013) calculated a nuclear bulk density which is in the range of 625–825 kg/m^3 or 513–713 kg/m^3. Our nuclear bulk density result of 540 ± 100 kg/m^3 falls in the latter density range.

6. 31P/Schwassmann-Wachmann 2: The determined nuclear bulk density of 490 ± 70 kg/m^3 is higher than the lower limit of 460 kg/m^3 estimated by Luu and Jewitt (1992).

7. 92P/Sanguin: The determined nuclear bulk density of 560 ± 110 kg/m^3 is higher than the lower limit of 490 kg/m^3 obtained by Snodgrass *et al.* (2005).

8. C/1983 H1 (IRAS-Araki-Alcock): The determined nuclear bulk density of 600 ± 100 kg/m^3 is higher than 450 kg/m^3 obtained by Harmon *et al.* (1999).

4. Discussion

We have obtained nuclear bulk densities (Tables 1 and 2) which are consistent with those in the literature (see Section 3). The uncertainties in nuclear bulk densities are mainly due to the uncertainties in the radii of the nuclei. We assume that the uncertainties in H are not greater than 1 mag and make a significant contribution to the uncertainties of the nuclear bulk densities. So we added these two contributions, *i.e.,* the uncertainty in the radii of the nuclei and the uncertainty of the absolute total magnitude of the comet $\Delta H = 1.0$ mag. For comet 81P/Wild 2 $\Delta H = 0.2$ mag (Ferrín 2009), the uncertainty of nuclear bulk density is 20 kg/m^3 (Table 1), while for C/1995 O1 (Hale-Bopp) $\Delta H = 0.1$ mag (Paradowski 1997), the uncertainty is 60 kg/m^3 (Table 2).

Since there is a lack of nuclear bulk density for most of the comets tested in the literature, we plotted in Fig. 4 the bulk density distribution of the 136 cometary nuclei, among which 96 nuclei belong to SP comets and 40 nuclei to LP comets with a period greater than 200 yr. To check whether the nuclear bulk density of the 136 comets is normally distributed, we performed the Shapiro-Wilk normality test at a significance level of 0.05, which rejects the null hypothesis that the sample comes from a normally distributed population because the p-value < 0.05. We can therefore assume that the bulk density distribution of cometary nuclei is related to their physical properties, such as size, shape, porosity and density. In Fig. 4, we see that the most likely density is in the range of 440–500 kg/m^3, and 80% of the 136 cometary nuclei have a bulk density $d < 600$ kg/m^3. The mean bulk density of this sample is 520 ± 10 kg/m^3. From 21 comets, Ferrín (2006) estimated a mean nuclear density of 520 ± 60 kg/m^3, which is consistent with our result for the 136 tested comets.

To find out whether the nuclear bulk density depends on the origin of comets, we selected from the 136 comets, 94 comets of the Jupiter-family (JF) and 40

Table 1

Parameters of the 96 SP comets

Comet	H [mag]	Ref. H	r_N [km]	Ref. r_N	d [kg/m^3]
1P/Halley	3.4	1	4.9 ± 0.1	2	540 ± 70
2P/Encke	7.5	1	2.4 ± 0.3	3	660 ± 110
4P/Faye	7.57	4	1.77 ± 0.04	5	590 ± 70
6P/d'Arrest	7.2	1	(1.817 ± 0.14) $1.52-1.70$, 2.23 ± 0.14	6, 7	570 ± 80
7P/Pons-Winnecke	5.5	8	2.64 ± 0.17	7	540 ± 80
8P/Tuttle	7.66	4	2.25 ± 0.5	9	650 ± 130
9P/Tempel 1	6.4	1	2.83 ± 0.1	10	620 ± 20^{a}
10P/Tempel 2	3.77	4	4.63 ± 0.3	11	550 ± 80
11P/Tempel-Swift-LINEAR	9.81	4	0.58 ± 0.04	7	490 ± 70
14P/Wolf	5.5	12a	(2.64 ± 0.15) 2.95 ± 0.19, 2.33 ± 0.12	7, 13	540 ± 80
15P/Finlay	8.96	4	0.92 ± 0.05	7	530 ± 70
16P/Brooks 2	9.0	4	0.72 ± 0.09	7	490 ± 80
17P/Holmes	5.94	4	1.71 ± 0.07	5	480 ± 60
19P/Borrelly	4.2	1	2.4 ± 0.1	14	440 ± 60
21P/Giacobini-Zinner	8.2	1	$[1.847\pm0.2]$ 3, 1 ± 0.2, 2.1	6, 15, 16	640 ± 100
22P/Kopff	5.91	4	(1.91 ± 0.17) 2.15 ± 0.17, 1.67 ± 0.18	7, 17	500 ± 80
29P/S-W 1	-0.44	4	15.58 ± 3.06	18	540 ± 100
31P/S-W 2	5.0	12b	(2.375 ± 0.12) 3.1, 1.65 ± 0.12	6, 7	490 ± 70
32P/Comas-Solá	6.5	12c	2.38 ± 0.23	7	580 ± 90
33P/Daniel	8.89	4	1.15 ± 0.09	7	580 ± 90
36P/Whipple	8.0	8	(2.09 ± 0.25) 1.9 ± 0.3, 2.28 ± 0.21	15, 19	660 ± 110
37P/Forbes	9.0	8	(1.02 ± 0.07) 0.81 ± 0.04, 1.23 ± 0.09	5, 7	560 ± 80
44P/Reinmuth 2	8.23	4	1.61 ± 0.07	5	610 ± 90
46P/Wirtanen	9.93	4	0.60 ± 0.02	11	510 ± 70
47P/Ashbrook-Jackson	4.21	4	(2.985 ± 0.145) 3.11 ± 0.21, 2.86 ± 0.08	7, 20	490 ± 70
50P/Arend	9.5	12d	(1.22 ± 0.08) 0.95 ± 0.03, 1.49 ± 0.13	5, 7	640 ± 90
51P-A/Harrington	11.64	4	0.41 ± 0.03	7	530 ± 80
52P/Harrington-Abell	8.86	4	1.3	11	600
56P/Slaughter-Burnham	8.5	8	$[1.648\pm0.14]$ 1.555, 1.92 ± 0.14, 1.5 ± 0.4	6, 7, 15	640 ± 100
59P/Kearns-Kwee	7.0	8	0.79 ± 0.03	5	400 ± 60
61P/Shajn-Shaldach	9.54	4	0.62 ± 0.02	20	490 ± 60
62P/Tsuchinshan 1	9.5	8	0.59 ± 0.09	7	480 ± 80
63P/Wild 1	9.18	4	1.46 ± 0.03	5	660 ± 80
67P/C-G	7.1	4	1.649 ± 0.007	21	532 ± 7^{b}
68P/Klemola	4.25	4	(2.5 ± 0.4) 2.8 ± 0.21, 2.2 ± 0.6	7, 15	450 ± 80
69P/Taylor	9.5	12e	0.87 ± 0.07	7	560 ± 80
70P/Kojima	6.4	4	1.83 ± 0.05	20	520 ± 70
71P/Clark	8.0	8	$[0.845\pm0.058]$ 1.305 ± 0.04, 0.68 ± 0.07	6, 11	460 ± 70
73P-C/S-W 3	11.63	4	0.41 ± 0.02	22	530 ± 70
76P/West-Kohoutek-Ikemura	12.17	4	0.30 ± 0.02	20	500 ± 70
77P/Longmore	7.22	4	1.66 ± 0.12	7	550 ± 80
78P/Gehrels 2	7.8	8	(1.38 ± 0.13) 1.35 ± 0.14, 1.41 ± 0.12	7, 19	550 ± 80
81P/Wild 2	5.8	1	2.00 ± 0.04	6	500 ± 20
84P/Giclas	8.5	8	0.90 ± 0.05	5	500 ± 70
86P/Wild 3	11.0	12f	0.41 ± 0.03	20	490 ± 70
92P/Sanguin	8.5	8	1.19 ± 0.22	6	560 ± 110
93P/Lovas 1	6.5	8	2.59 ± 0.26	7	600 ± 90
97P/Metcalf-Brewington	5.5	12g	1.7 ± 0.4	11	450 ± 100
101P/Chernykh	9.4	23	0.98 ± 0.09	7	580 ± 90
102P/Shoemaker 1	12.16	4	0.208 ± 0.012	18	430 ± 60
103P/Hartley 2	9.35	4	0.57 ± 0.08	24	460 ± 80
106P/Schuster	9.0	8	0.94 ± 0.03	5	540 ± 70
110P/Hartley 3	6.0	4	2.15 ± 0.04	20	530 ± 70
114P/Wiseman-Skiff	9.0	8	0.78 ± 0.04	11	500 ± 70
115P/Maury	9.51	4	1.108 ± 0.04	6	610 ± 80

Table 1

Concluded

Comet	H [mag]	Ref. H	r_N [km]	Ref. r_N	d [kg/m^3]
118P/Shoemaker-Levy 4	8.2	8	1.30 ± 0.22	7	560 ± 100
119P/Parker-Hartley	8.3	4	0.98 ± 0.11	7	510 ± 80
120P/Mueller 1	10.62	4	0.77 ± 0.05	25	610 ± 90
121P/Shoemaker-Holt 2	5.04	4	[2.17 ± 0.44] 3.87 ± 0.27, 1.62 ± 0.57	7, 13	470 ± 90
123P/West-Hartley	5.91	4	2.18 ± 0.23	7	520 ± 80
124P/Mrkos	7.55	4	[1.886 ± 0.3] 2.62 ± 0.16, 1.6 ± 0.4	7, 15	600 ± 110
126P/IRAS	7.88	4	1.57 ± 0.14	26	580 ± 90
127P/Holt-Olmstead	9.5	23	0.90 ± 0.07	7	560 ± 80
128P/Shoemaker-Holt 1	7.1	23	(2.26 ± 0.14)	19	610 ± 90
131P/Mueller 2	9.3	8	(0.99 ± 0.07) 1.11 ± 0.09, 0.87 ± 0.04	7, 25	570 ± 80
132P/Helin-Roman-Alu 2	7.2	8	0.81 ± 0.05	7	410 ± 60
139P/Väisälä-Oterma	7.2	23	1.37 ± 0.08	7	510 ± 70
144P/Kushida	8.5	12h	0.84 ± 0.05	7	490 ± 70
146P/Shoemaker-LINEAR	9.5	8	0.96 ± 0.08	7	580 ± 90
148P/Anderson-LINEAR	9.0	12i	1.14 ± 0.07	7	580 ± 80
149P/Mueller 4	8.0	12j	1.42 ± 0.10	7	570 ± 80
152P/Helin-Lawrence	7.76	4	1.03 ± 0.11	7	480 ± 80
159P/LONEOS	9.21	4	1.46 ± 0.23	7	660 ± 120
160P/LINEAR	8.3	8	(0.93 ± 0.08) 0.99 ± 0.09, 0.87 ± 0.07	7, 25	500 ± 80
168P/Hergenrother	10.51	4	0.48 ± 0.04	7	500 ± 70
171P/Spahr	9.5	8	1.25 ± 0.09	7	640 ± 90
189P/NEAT	17	8	0.064 ± 0.009	18	480 ± 80
213P/Van Ness	8.38	4	1.53 ± 0.29	7	610 ± 120
215P/NEAT	5.94	4	1.29 ± 0.08	7	430 ± 60
219P/LINEAR	7.3	8	1.00 ± 0.06	7	450 ± 60
221P/LINEAR	9.0	8	1.03 ± 0.06	7	560 ± 80
228P/LINEAR	9.7	23	1.23 ± 0.17	7	650 ± 110
237P/LINEAR	7.43	4	1.03 ± 0.17	27	470 ± 90
246P/NEAT	3.14	4	4.20 ± 0.43	7	490 ± 80
252P/LINEAR	10.86	4	0.30 ± 0.03	28	430 ± 70
260P/McNaught	11.65	4	[0.59 ± 0.05] 1.54 ± 0.09, 0.322, 0.414 ± 0.039	7, 18, U	620 ± 90
315P/LONEOS	9.17	4	[1.124 ± 0.058] 5.41 ± 0.31, 0.512 ± 0.025	7, 18	590 ± 80
344P/Read	5.5	8	1.41 ± 0.11	7	420 ± 60
356P/WISE	6.5	23	1.265 ± 0.445	27	450 ± 120
P/2004 A1 (LONEOS)	6.5	12k	3.49 ± 0.28	7	680 ± 100
P/2004 DO29 (S-L)[c]	10.0	8	1.07 ± 0.09	7	640 ± 100
P/2004 V3 (Siding Spring)	9.7	8	0.92 ± 0.08	7	580 ± 90
P/2004 V5-A (LINEAR-Hill)	8.3	8	1.66 ± 0.24	7	620 ± 110
P/2005 W3 (Kowalski)	8.5	23	1.16 ± 0.27	7	550 ± 120
P/2005 XA54 (LONEOS-Hill)	5.5	8	2.71 ± 0.17	7	540 ± 80
P/2013 J2 (McNaught)	9.13	4	0.495 ± 0.027	18	430 ± 60

The values of r_N in brackets are the mean of the following values, while those in square brackets are the effective radii of the volume-equivalent sphere calculated from the following values, so those in brackets are taken for determining the nuclear density.

[a] A'Hearn *et al.* (2005), [b] Jorda *et al.* (2016), [c] Spacewatch-LINEAR.

References: 1 Ferrín (2009), 2 Keller *et al.* (1994), 3 Fernández *et al.* (2000), Boehnhardt *et al.* (2008), 4 COBS, 5 Lamy *et al.* (2009), 6 Meech *et al.* (2004), 7 Fernández *et al.* (2013), 8 Yoshida (2020), 9 Harmon *et al.* (2008, 2010), 10 Thomas *et al.* (2013a), 11 Lamy *et al.* (2004), 12a MPC *https://minorplanetcenter.net/* 100283, 12b MPC 108597, 12c MPC 92276, 12d MPEC *https://minorplanetcenter.net/* 2015-S97, 12e MPC 114607, 12f MPC 109147, 12g MPC 77020, 12h MPC 104795, 12i MPEC 2014-R70, 12j MPC 75720, 12k MPC 82312, 13 Lowry *et al.* (2003a), 14 Soderblom *et al.* (2002), 15 Tancredi *et al.* (2000), 16 Mueller (1992), 17 Lamy *et al.* (2002), 18 Paradowski (2020), 19 Lowry and Weissman (2003b), 20 Lamy *et al.* (2011), 21 Jorda *et al.* (2016), 22 Toth and Lisse (2006), 23 JPL *https://ssd.jpl.nasa.gov/sbdb.cgi*, 24 Lisse *et al.* (2009), Thomas *et al.* (2013b), 25 Snodgrass *et al.* (2008), 26 Groussin *et al.* (2004), 27 Bauer *et al.* (2015), 28 Li *et al.* (2017), 29 Hanner *et al.* (1987), 30 Sosa and Fernández (2011), 31 Paradowski (1997), 32 McCarthy *et al.* (2007), 33 Lisse *et al.* (1999), 34 Harmon *et al.* (1999), 35 Farnham *et al.* (2001), 36 Sekanina and Chodas (2012), 37 Scarmato *https://www.researchgate.net/publication/263078578*, U Paradowski (private communication).

Table 2

Parameters for the 40 LP comets

Comet	H [mag]	Ref. H	r_N [km]	Ref. r_N	d [kg/m^3]
C/1983 H1 (IRAS-Araki-Alcock)	7.8	1	1.75 ± 0.25	6	600 ± 100
C/1983 J1 (Sugano-Saigusa-Fujikawa)	10.5	1	0.37 ± 0.05	29	450 ± 80
C/1985 R1 (Hartley-Good)	8.3	30	0.88 ± 0.05	30	490 ± 70
C/1989 Q1 (Okazaki-Levy-Rudenko)	7.5	30	1.00 ± 0.12	30	460 ± 80
C/1993 Y1 (McNaught-Russell)	8.7	30	0.75 ± 0.02	30	480 ± 60
C/1995 O1 (Hale-Bopp)	-1.3	31	20 ± 5	32	540 ± 60
C/1995 Y1 (Hyakutake)	7.3	30	1.14 ± 0.02	30	480 ± 60
C/1996 B2 (Hyakutake)	4.3	1	2.4 ± 0.5	33	450 ± 90
C/1998 K5 (LINEAR)	12.7	4	0.55	34	680
C/1999 J3 (LINEAR)	8.4	30	0.71 ± 0.01	30	450 ± 60
C/1999 S4 (LINEAR)	9.23	4	0.44	35	410
C/2002 V1 (NEAT)	6.1	30	1.57 ± 0.16	30	470 ± 70
C/2004 Q2 (Machholz)	5.2	30	1.82 ± 0.17	30	450 ± 70
C/2005 W2 (Christensen)	10.3	8	0.52 ± 0.09	7	500 ± 90
C/2006 S3 (LONEOS)	2.19	4	5.019 ± 0.385	18	470 ± 70
C/2007 F1 (LONEOS)	8.0	30	0.75 ± 0.10	30	440 ± 70
C/2007 W1 (Boattini)	8.5	30	0.65 ± 0.03	30	440 ± 60
C/2009 P1 (Garradd)	3.68	4	2.945 ± 0.113	18	450 ± 60
C/2010 FB87 (WISE-Garradd)	6.7	8	2.44 ± 0.56	27	600 ± 120
C/2010 S1 (LINEAR)	1.25	4	5.438 ± 0.247	18	430 ± 60
C/2010 X1 (Elenin)	9.97	4	0.430 ± 0.029	18	450 ± 60
C/2011 J2 (LINEAR)	6.17	4	1.298 ± 0.060	18	440 ± 60
C/2011 L4 (PANSTARRS)	5.1	4	2.317 ± 0.190	18	490 ± 70
C/2011 W3 (Lovejoy)	10.0	36	0.455 ± 0.104	18	460 ± 90
C/2012 F6 (Lemmon)	4.8	4	2.151 ± 0.133	18	460 ± 60
C/2012 J1 (Catalina)	6.77	4	1.285 ± 0.059	18	470 ± 60
C/2012 K1 (PANSTARRS)	4.7	4	2.352 ± 0.130	18	470 ± 70
C/2012 S1 (ISON)	8.1	4	0.828 ± 0.110	18	460 ± 80
C/2013 A1 (Siding Spring)	8.8	4	0.435 ± 0.077	37	390 ± 80
C/2013 R1 (Lovejoy)	7.04	4	1.266 ± 0.100	18	480 ± 70
C/2013 US10 (Catalina)	5.64	4	1.645 ± 0.239	18	450 ± 80
C/2014 B1 (Schwartz)	0.92	4	6.364 ± 0.214	18	440 ± 60
C/2014 E2 (Jacques)	6.8	4	1.090 ± 0.038	18	440 ± 60
C/2014 Q2 (Lovejoy)	2.42	4	4.314 ± 0.227	18	450 ± 60
C/2015 F4 (Jacques)	6.46	4	1.205 ± 0.108	18	440 ± 70
C/2015 V2 (Johnson)	6.0	4	1.700 ± 0.138	18	480 ± 70
C/2015 ER61 (PANSTARRS)	7.06	4	0.936 ± 0.090	18	430 ± 70
C/2015 VL62 (Lemmon-Yeung-PANSTARRS)	8.5	4	0.848 ± 0.033	18	490 ± 70
C/2016 A8 (LINEAR)	8.28	4	0.877 ± 0.119	18	480 ± 80
C/2017 O1 (ASASSN)	7.55	4	0.919 ± 0.092	18	450 ± 70

The references are listed in Table 1.

LP comets, and then plotted their density distributions in Figs. 5 and 6, respectively. We use the Shapiro-Wilk normality test for both nuclear density distributions, which also rejects the null hypothesis as the p-value < 0.05 for each distribution, *i.e.,* the samples do not come from a population with normal distribution. This confirms that the nuclear density distribution of both samples is related to their physical properties. In addition, we performed the Kolmogorov-Smirnov test of density distribution for both samples, *i.e.,* 40 LP and 94 JF comets, which yields 95% confidence that the densities come from different distributions. In Fig. 5,

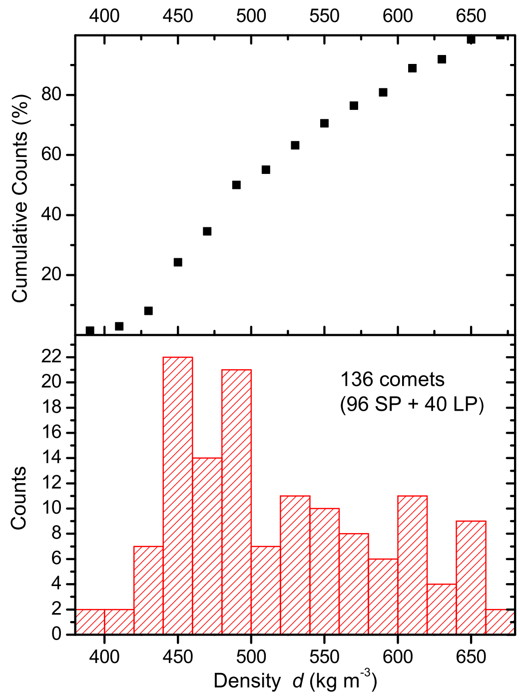

Fig. 4. Histogram and cumulative bulk density distribution of the 136 cometary nuclei, among which 96 nuclei belong to the SP comets listed in Table 1, and 40 nuclei belong to the LP comets listed in Table 2.

we see that the most likely nuclear density of the 94 JF comets is in the range of 480–500 kg/m^3, and 80% of these comets have a nuclear density $d < 600$ kg/m^3. The mean nuclear bulk density of this sample is 540 ± 10 kg/m^3, and is consistent with JF comet 67P/Churyumov-Gerasimenko density of 532 ± 7 kg/m^3 (Jorda *et al.* 2016) within the uncertainties. This proves the correctness of our method of determining the bulk density of cometary nuclei. The most likely nuclear density of the 40 LP comets is in the range of 440–460 kg/m^3, and 90% of these comets have a nuclear density $d < 500$ kg/m^3 (see Fig. 6). The mean nuclear bulk density of this sample is 470 ± 10 kg/m^3, and is much lower than the 94 JF comets of 540 ± 10 kg/m^3.

The studied population of 136 comets shows that the nuclei of SP comets have a higher bulk density than that of LP comets. Thus, we have shown the dependence of the nuclear bulk density on the origin of comets. Bauer *et al.* (2017) found a significant difference in the mean size of the nucleus of LP comets compared to SP comets with a ratio of 1.6. This can be explained by the fact that the nuclei of LP comets that came from the Oort Cloud contain the primordial material of the Solar System and are the most pristine samples, while the SP comets formed through the processes of disintegration and evolutionary mass loss leading to smaller sizes (Bauer *et al.* 2017). Therefore, we suppose that the above processes led to higher nuclear densities of the SP comets.

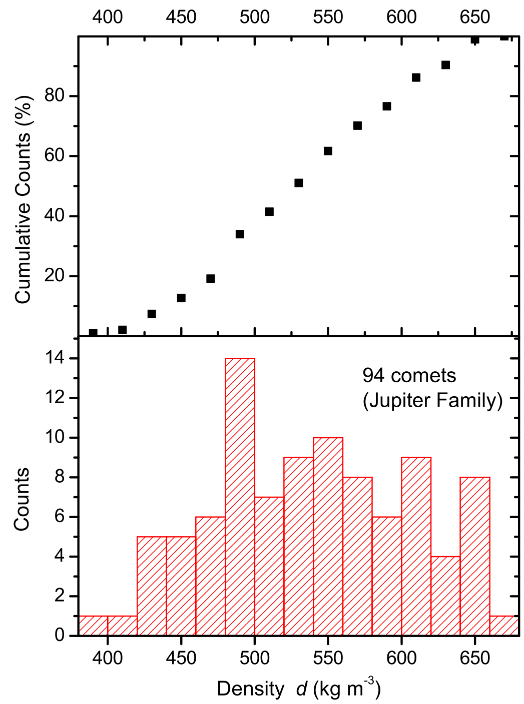

Fig. 5. Histogram and cumulative bulk density distribution of the 94 cometary nuclei, which belong to the JF comets and are included in Table 1 (comets 1P/Halley and 2P/Encke do not belong to the Jupiter family).

Below, we discuss the comets with nuclear bulk density higher than a lower limit. The lower limit on the nuclear bulk density d [g/cm^3] is given by (Pravec and Harris 2000):

$$d = \frac{10.9}{P^2} \times \frac{a}{b}, \tag{6}$$

where P is the rotation period in hours and a/b is the axis ratio of the nucleus. According to the above formula, the nuclear bulk density d must be sufficient to prevent rotational break up due to centrifugal forces. Using Eq.(6), we calculate the lower limit on the bulk density of comet nuclei whose rotation period is less than 10 hours, and compare with the determined nuclear density (Table 1).

1. 6P/d'Arrest: The determined nuclear bulk density of 570 ± 80 kg/m^3 is higher than the lower limit of 265 kg/m^3 calculated from Eq.(6) with the period of the nuclear rotation of 6.67 h (Gutiérrez *et al.* 2003) and $a/b = 1.08$ (Kokotanekova *et al.* 2017).

2. 7P/Pons-Winnecke: The determined nuclear bulk density of 540 ± 80 kg/m^3 is higher than the lower limit of 227 kg/m^3 ($P = 7.9$ h and $a/b = 1.3$, Snodgrass *et al.* 2005).

3. 10P/Tempel 2: The determined nuclear bulk density of 550 ± 80 kg/m^3

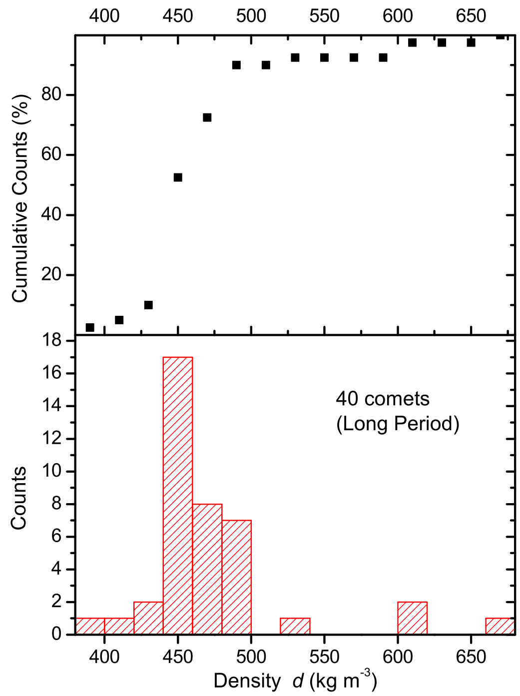

Fig. 6. Histogram and cumulative bulk density distribution of the 40 cometary nuclei, which belong to LP comets listed in Table 2.

is higher than the lower limit of 259 kg/m^3 calculated using $P = 8.948$ h (Schleicher *et al.* 2013) and $a/b = 1.9$ (Jewitt and Luu 1989).

4. 14P/Wolf: The determined nuclear bulk density of 540 ± 80 kg/m^3 is higher than the lower limit of 189 kg/m^3 ($P = 9.02$ h and $a/b = 1.41$, Kokotanekova *et al.* 2017).

5. 17P/Holmes: The determined nuclear bulk density of 480 ± 60 kg/m^3 is higher than the lower limit of 273 kg/m^3 ($P = 7.2$ h and $a/b = 1.3$, Snodgrass *et al.* 2006).

6. 21P/Giacobini-Zinner: The determined nuclear bulk density of 640 ± 100 kg/m^3 is higher than the lower limit of 362 kg/m^3 calculated using $P = 9.5$ h (Leibowitz and Brosch 1986) and $a/b = 3$ (assumed from Table 1).

7. 31P/Schwassmann-Wachmann 2: The determined nuclear bulk density of 490 ± 70 kg/m^3 is lower than the lower limit of 560 kg/m^3 calculated using $P = 5.58$ h and $a/b = 1.6$ (Luu and Jewitt 1992). The comet is a candidate for future splitting (Kokotanekova *et al.* 2017).

8. 46P/Wirtanen: The determined nuclear bulk density of 510 ± 70 kg/m^3 is higher than the lower limit of 424 kg/m^3 calculated using $P = 6.0$ h (Lamy *et al.* 1998) and $a/b = 1.4$ (Boehnhardt *et al.* 2002).

9. 61P/Shajn-Shaldach: The determined nuclear bulk density of 490 ± 60 kg/m^3 is lower than the lower limit of 576 kg/m^3 ($P = 4.9$ h and $a/b = 1.27$, Lamy *et al.* 2011).

10. 73P-C/Schwassmann-Wachmann 3: The determined nuclear bulk density of 530 ± 70 kg/m^3 is much lower than the lower limit of 1697 kg/m^3 calculated with $P = 3.4$ h (Drahus *et al.* 2010) and $a/b = 1.8$ (Toth and Lisse 2006). This confirms that the comet's ongoing splitting is due to rotational instabilities (Kokotanekova *et al.* 2017).

11. 76P/West-Kohoutek-Ikemura: The determined nuclear bulk density of 500 ± 70 kg/m^3 is higher than the lower limit of 363 kg/m^3 ($P = 6.6$ h and $a/b = 1.45$, Lamy *et al.* 2011).

12. 92P/Sanguin: The determined nuclear bulk density of 560 ± 110 kg/m^3 is higher than the lower limit of 479 kg/m^3 ($P = 6.22$ h and $a/b = 1.7$, Snodgrass *et al.* 2005).

13. 260P/McNaught: The determined nuclear bulk density of 620 ± 90 kg/m^3 is higher than the lower limit of 606 kg/m^3 calculated with $P = 8.16$ h (Manzini *et al.* 2014) and $a/b = 3.7$ (taking $a = 1.54$ km and $b = 0.414$ km from Table 1).

Weissman *et al.* (2004) found that the bulk density of cometary nuclei is in the range of 500–1200 kg/m^3, with the "best" value 600 kg/m^3. However, Kokotanekova *et al.* (2017) confirmed the known cut-off in bulk densities at about 600 kg/m^3 assuming the nuclei of JF comets are strength-less. This supports our results.

We have not included comets belonging to the Centaurs, asteroids, interstellar and TNO in Tables 1 and 2, so we determine their bulk density using Eqs.(1), (3) and (5), and list them in Table 3.

95P (2060 Chiron) is a centaur with a diameter of 166 km (JPL) and the absolute total magnitude $H = -1.11$ (COBS). Using Eqs.(1), (3) and (5), we obtain the bulk density $d = 960$ kg/m^3 which is consistent with the upper limit of 1000 kg/m^3 estimated by Meech *et al.* (1997).

107P (4015 Wilson-Harrington) is an asteroid (Apollo group) with the absolute magnitude $H = 12.9$ (Ferrín 2009) and the radius of the equivalent volume sphere of 1.713 ± 0.04 km, calculated from the literature values of the radii equal to 1.96 ± 0.02 km (Meech *et al.* 2004), 1.77 ± 0.03 km (Lowry and Weissman 2003b), and 1.45 ± 0.10 km (Fernández *et al.* 2013). Finally, we obtain a bulk density of 1080 ± 140 kg/m^3 which is higher than the lower limit of 360 kg/m^3 calculated with $P = 6.10$ h (Osip *et al.* 1995) and $a/b = 1.23$ (Lowry and Weissman 2003b) in Eq.(6).

133P/Elst-Pizarro belongs to the Main-belt asteroids (MBA) that has a diameter of 4.6 ± 0.6 km (Hsieh *et al.* 2009) and the absolute magnitude $H = 15.6$ (JPL).

Table 3

Parameters for centaurs, asteroids, interstellar and trans-Neptunian objects (TNO)

Comet	H [mag]	Ref. H	$r_{\rm N}$ [km]	Ref. $r_{\rm N}$	d [kg/m^3]
95P (2060 Chiron) centaur	-1.11	1	83	2	960
107P (4015 Wilson-Harrington) *asteroid*	12.9	3	1.713 ± 0.04	this work	1080 ± 140
133P/Elst-Pizarro asteroid (MBA)	15.6	2	2.3 ± 0.3	4	1670 ± 280
174P (60558 Echeclus) centaur	3.68	1	29.5 ± 2	2	1120 ± 160
1996 PW trans-Neptunian Object (TNO)	13.9	2	4	5	1700
C/2001 OG108 (LONEOS) asteroid (damocloid)	8.7	3	8.9 ± 0.7	6	1260 ± 190
118401 LINEAR (1999 RE70) asteroid (MBA)	15.2	2	1.75 ± 0.05	2	1430 ± 180
45 Eugenia asteroid (MBA)	7.5	2	101.163 ± 1.084	2	2850 ± 360
253 Mathilde asteroid (MBA)	10.3	2	26.4	2	2340
433 Eros asteroid (Amor group)	11.16	2	8.42 ± 0.03	2	1650 ± 200
2I/Borisov interstellar	10.5	7	0.5	8	504

References: 1 COBS, 2 JPL, 3 Ferrín (2009), 4 Hsieh *et al.* (2009), 5 Weissman and Levison (1997), 6 Abell *et al.* (2003), 7 Yoshida (2020), 8 Jewitt *et al.* (2020).

We obtain a bulk density of 1670 ± 280 kg/m^3 which is higher than the lower limit of 1300 kg/m^3 estimated by Hsieh *et al.* (2004, 2010).

253 Mathilde is an asteroid (MBA) with a diameter of 52.8 km and the absolute magnitude $H = 10.3$ (JPL), and we obtain a bulk density of 2340 kg/m^3. However, Veverka *et al.* (1997) obtained a much lower density of 1300 kg/m^3.

433 Eros is an asteroid (Amor group) with a diameter of 16.84 ± 0.06 km and the absolute magnitude $H = 11.16$ (JPL), and we obtain a bulk density of 1650 ± 200 kg/m^3. However, Yeomans *et al.* (2000) and Miller *et al.* (2002) obtained a much higher density of 2670 ± 30 kg/m^3.

2I/Borisov is an interstellar comet with a nucleus diameter of 1 km (Jewitt *et al.* 2020) and the absolute total magnitude (coma plus nucleus) $H = 10.5$ (Yoshida 2020). The determined nuclear density of 504 kg/m^3 is consistent with 500 kg/m^3 estimated by Jewitt *et al.* (2020).

The discrepancy of the bulk densities of the asteroids 253 Mathilde and 433 Eros with the literature values can be explained by the limitation of the accuracy of the calibration function (Eq. 5), which serves well for comet densities lower than 1000 kg/m^3, while for asteroids a new calibration function should be plotted using a range of standard densities higher than 1000 kg/m^3. Nevertheless, our calibration function paves the way for measuring the bulk density of cometary nuclei. Thus, we have shown that the bulk density of cometary nuclei can be determined on the basis of the calibration function (Eq. 5), effective nuclear radii $r_{\rm N}$, and absolute total magnitudes (coma + nucleus) H of comets.

5. Summary and Conclusions

From the 136 comets tested by the new indirect method of determining the bulk density of their nuclei, we can draw the following conclusions:

(i) The new indirect method uses the calibration function with Eq.(5) in the form $a_1 = 10.88526 + 3.39275 \times a_2$ and equation $\log D_{\mathrm{N}} = a_2 - 0.13H$, where D_{N} is the nucleus' effective diameter [km], H is the comets' absolute total magnitude (coma + nucleus). To obtain a nuclear bulk density [kg/m^3] the formula $d = \frac{6}{\pi} \times 10^a$ where $a = a_1 - 3a_2 - 9$ and $\pi \approx 3.14$ is used (Paradowski 2020).

(ii) The basis of the indirect method is the above calibration function (Eq. 5 and Fig. 3) plotted from the known nuclear bulk densities and sizes, and absolute total magnitudes (coma + nucleus) of comets 9P/Tempel 1, 67P/Churyumov-Gerasimenko, 81P/Wild 2 and the mean nuclear bulk density of 31 comets determined in the previous work (Paradowski 2020). These four data points in the graph a_1 to a_2 lie on a straight line with Eq.(5) and the correlation coefficient $k = 0.99996$ which shows a very good fit to the four data points (Fig. 3). This proves that our indirect method for determining the bulk density of cometary nuclei has a high accuracy of 1.5%. However, we note that this accuracy is limited to a density lower than 1000 kg/m^3 due to the narrow standard density range of 453–620 kg/m^3 on the basis of which the calibration function is plotted.

(iii) We note that the comets' absolute total magnitude (coma + nucleus) H is very important as it plays the main role in the preparation of the calibration function (Eq. 5) and the determination of the bulk density of cometary nuclei (see Eqs. 2 and 3). We obtained the H values from the (COBS), and also some of them came from Seiichi Yoshida's Home Page[4], the JPL Small-Body Database Browser (JPL), and the Minor Planet Center (MPC).

(iv) Our method also allows one to determine the bulk density of the nuclei of SP, LP, or hyperbolic comets. For example, the nuclear bulk density of the SP comet 6P/d'Arrest is 570 ± 80 kg/m^3, the nuclear bulk density of the LP comet C/2011 L4 (PANSTARRS) is 490 ± 70 kg/m^3, and the nuclear bulk density of the hyperbolic comet 2I/Borisov is 504 kg/m^3.

(v) We have obtained the nuclear bulk densities of comets, the accuracy of which depends mainly on the uncertainty of the measurement of the nuclear radii (see Tables 1, and 2), as well as on the uncertainty of the absolute total magnitude H (assumed $\Delta H = 1$ mag). For example, the nuclear bulk densities

[4] *www.aerith.net*

of comets 1P/Halley, 2P/Encke, and C/1999 J3 (LINEAR) are 540 ± 70, 660 ± 110, and 450 ± 60 kg/m^3, respectively.

(vi) The mean nuclear bulk density of the studied 136 comets is $d = 520 \pm 10$ kg/m^3 with an uncertainty of 2%, which proves the correctness of our indirect method of determining the nuclear bulk density.

(vii) We have also obtained the mean nuclear bulk density of 540 ± 10 kg/m^3 from the 94 JF comets, which is consistent with comet 67P/Churyumov-Gerasimenko density of 532 ± 7 kg/m^3 (Jorda *et al.* 2016) within the uncertainties. Comet 67P/Churyumov-Gerasimenko belongs to the Jupiter family and supports our result.

(viii) We have found the relationship of the nuclear bulk density to the origin of the comets. For the 40 LP comets the mean bulk density is 470 ± 10 kg/m^3, whereas for the 94 SP comets from the Jupiter family it is 540 ± 10 kg/m^3. The lower bulk density of the nuclei of LP comets than that of SP comets may help in studying the structure and evolution of the Solar System.

(ix) The proposed indirect method can be applied to comets observed in the future and those stored in archives, provided that their effective nuclear radii $r_{\rm N}$ and absolute total magnitudes (coma + nucleus) H are accurately determined.

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